Teacher notes
Topic E

A few results on the photoelectric effect.

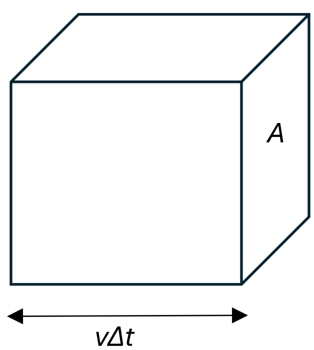
Suppose that *N* photons per unit time per unit area are incident normally on a photo-surface of area *A*. Assuming that each photon causes the emission of one electron, what is the current leaving the surface?

The obvious answer is this: in a time of Δ*t* seconds the number of photons incident on the entire surface is $NA\Delta t$ and so the charge leaving the surface is $\Delta Q = eNA\Delta t$. Hence the current is $i = \frac{\Delta Q}{\Delta t} = eNA$.

The point is that the current is independent of the speed with which the electrons leave the surface. This may sound surprising to anyone familiar with the current formula $i = enAv$ where *n* is the number of electrons per unit volume and *v* is the (drift) speed of the electrons. But we can show that the two formulae are in fact equivalent.

The number of electrons emitted in time Δ*t* seconds is *NA*Δ*t*. These electrons will occupy a volume *Av*Δ*t* and so the number of electrons per unit volume is $n = \frac{NA\Delta t}{Av\Delta t} = \frac{N}{v}$. From $i = enAv$ we get

$i = e\frac{N}{v}Av = eNA$ just as before; the speed cancels out.



We can also estimate the pressure exerted on the surface by the incident photons. Assuming again that each photon is incident normally and bounces off the surface with no change in wavelength we have that:

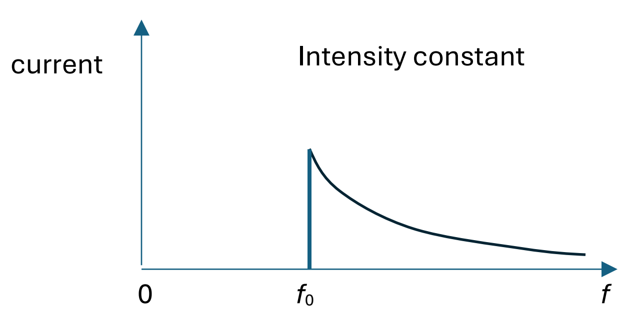
The change in momentum of one photon is $\frac{2h}{\lambda}$. In time Δ*t* seconds the number of photons incident on the entire surface is $NA\Delta t$ and so the total change of photon momenta is $NA\Delta t \times \frac{2h}{\lambda}$. The force on the surface is the rate of change of momentum and so $F = \frac{NA\Delta t \times \frac{2h}{\lambda}}{\Delta t} = 2NA\frac{h}{\lambda}$. Hence the pressure exerted on the surface is $P=\frac{F}{A}=2N\frac{h}{\lambda}$. We can express the pressure in terms of the intensity of the radiation incident on the surface: the energy of one photon is $\frac{hc}{\lambda}$ and so the power incident (energy per unit time) is $NA\frac{hc}{\lambda}$; hence the intensity is $I=\frac{NA\frac{hc}{\lambda}}{A}=\frac{Nhc}{\lambda}$. This means we can write $N=\frac{I\lambda}{hc}$ and so $P=2\frac{I\lambda}{ch}\frac{h}{\lambda}$ which simplifies to

$$P=\frac{2I}{c}$$

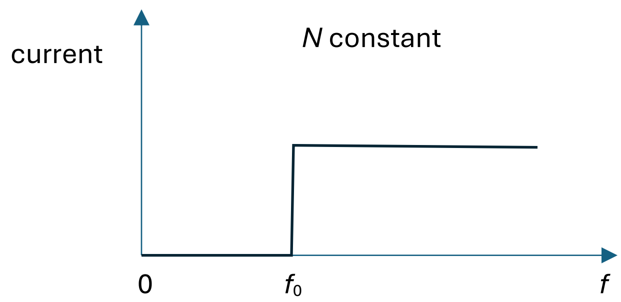
It is interesting that the Planck constant has cancelled out. This means that this formula also holds classically i.e. we do not need photons and quantum theory. Indeed, the same formula can be derived using classical electromagnetism, light exerts pressure on the surface independently of whether we treat light as particles or a wave.

Making use of $N=\frac{I\lambda}{hc}$ we can rewrite the formula for the current leaving the surface that we derived above: $i=eNA$ and so $i=\frac{eI\lambda A}{hc}=\frac{eIA}{hf}$. This shows that if we increase the frequency keeping the intensity constant, the current decreases. This, of course, under the assumption that **each** photon causes the emission of **one** electron. This result is heavily dependent on this assumption. In fact, not all photons cause electrons to be emitted, some photons are reflected! And furthermore, the fraction that get reflected depends on the wavelength so changing the wavelength without knowing what happens to the reflected fraction greatly complicates the problem.

So, if the intensity is kept constant, then from it follows that the current decreases as the frequency increases:



If instead the number of photons per unit time per unit area *N* is constant, from $i = eNA$ we see that the current is constant independently of frequency. This leads to the following graph where f_0 is the threshold frequency:



But, as mentioned earlier, both graphs are subject to the assumption that every photon causes the emission of one electron. So, these are approximately correct at the IB level but not so in real Physics.